A Generalized Approach to International Comparison of Agricultural Output and Productivity

by

A. Maddison and D.S. Prasada Rao[1]

This paper has three purposes. It is intended to round off the results of the Maddison and van Ooststroom (1995) study by using their data set to provide a fuller range of results showing Paasche, Laspeyres and Geary-Khamis versions of their results. It is intended to demonstrate the techniques for and the advantages of expanding future FAO studies to include Laspeyres, Paasche and Fisher variants as well as Geary-Khamis results. Thirdly, it is intended to provide a generalised view of how the techniques hitherto used by the ICOP project and FAO can be integrated with the approach used by ICP and EUROSTAT - in particular the methods by which price holes in data can be filled by the CPD (country-product-dummy) technique, and how Geary-Khamis purchasing power parities (PPPs) can be derived.

I Our Data Set and Estimation Procedure

For illustrative purposes we have used the 12 country data set in Maddison and van Ooststroom (1995). Maddison and van Ooststroom used a simple aggregation procedure for farm output, using FAO price and quantity data for 1975 in the form in which they were available in 1984. There have been subsequent amendments in FAO's basic price and quantity estimates, but as they were not available when we wrote, and did not seem to be of great significance, we ignored these subsequent modifications in the data set.

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Commodity Production Tables

Table B.1 shows FAO estimates of quantities produced by farmers. Table B.2 shows quantities of feed and seed inputs (including imported products). Table B.3 shows commodity output net of feed and seed.

We took an array of 111 products from the Maddison-van Ooststroom (1995) data set. Thus there are 111 potential items in our production array in Table B.1. For the 12 countries this means a potential 1332 entries. In fact there were 585 holes in this array, because of the items which were not produced or not recorded as produced in most countries. Thus we had 747 quantity entries. The country with the most entries was the USA with 84 items. The country with the least was the UK, which with its narrower band of climatic conditions, and a long history of agricultural specialization, had only 43 items.

Prices and PPPs

In the price file there were 719 holes, i.e. we had 613 price entries. There were cases where we had prices where there was no production in the country concerned, so there were in fact only 545 cases where our data set permitted matching and multiplication of price and quantity to arrive at the estimate of value shown in Table C.1C. There were 202 price holes which we set out to fill by using the CPD technique. Table C.1D shows that the proportionate importance of the price holes in value terms was not as big as their absolute numbers suggest.

Maddison and van Ooststroom plugged the price holes mainly by the shadow price method i.e. they used the relationship of the missing price to the wheat (or rice) price in another country and used this coefficient to arrive at their shadow price. In some case they used proxy prices. Partly as a simplification, partly because of analytic preference, they expressed all the values in US (plus shadow) prices (i.e. in "Paasche" prices, using the jargon of ICP). These value estimates are contained in Table C.1B. In order to derive purchasing power parities (PPPs) they also established shadow prices for each country in order to estimate the value of output for all items produced in national currencies. In fact there were four price holes they did not fill, i.e. for treenuts, two kinds of processed sugar and meat, NES (not elsewhere specified).

As we needed to get a full array of results to estimate output variants in terms of Paasche, Laspeyres, Fisher and Geary-Khamis prices, we needed to fill all the price gaps, and fill them more systematically than was possible with the shadow price procedure.

Our procedure involved the same two steps that were used by Kravis, Heston and Summers in ICP III, i.e. the CPD technique of filling all the gaps in prices for individual items and the Geary-Khamis technique for estimating international prices. The CPD is based on probability (stochastic) assumptions about (a) the characteristics of commodity price structures, averaged across countries, and (b) the characteristics of each country's general price levels. Our CPD technique used only the array of prices we had from FAO for the 12 countries, except for 7 cases where there was no price for any item in any country. Unless there is a price for a commodity in at least one country, CPD cannot fill the price gap, so for 7 items we inserted prices, i.e. for poultry we used the chicken prices of FAO and for raisins we used the price of grapes; for animal skins we inserted the three proxy prices of Maddison and van Ooststroom, as if they were US prices. For cottonseed and cotton lint we used national US sources. The CPD results can be seen in Table A.2 where the price array is complete for all 1332 entries.

The last column of Table A.2 contains the Geary-Khamis international prices which were computed using all the quantitative and price information at our disposal (independently of the CPD procedure, which, as already mentioned, makes no use of quantitative information). The CPD and Geary-Khamis techniques are described in separate notes.

In all cases we used our producer prices including the CPD estimates to value inputs of feed and seed. In Table B.2, one can see that the number of feed and seed items is much smaller than 747 entries we had for production.

Table A.3 shows the Paasche PPPs. For individual items these are simply the ratio of prices of the relevant items in a given country relative to the US prices. Thus the Argentine PPP for wheat is the Argentine wheat price from Table A.2 of 1472 pesos divided by the US price of $138.01, giving a PPP of 10.645 pesos to the dollar. For the group totals (cereals etc.) which correspond to the ICP notion of a "basic heading", the PPP is a weighted average using the item PPPs within the heading and the quantity weights for each country from Table B.1. The zero entries in the table are for commodities where there is no production of the relevant item in the country concerned, which consequently get a zero weight in the calculation of the PPP at the basic heading level.

Table A.4 shows the Laspeyres PPPs. For individual items all the PPP ratios are the same as in Table A.3, with the difference that a zero PPP is shown for those items where no production was recorded in the USA. The Laspeyres PPP's are different from the Paasche PPPs at the basic heading level, because the Laspeyres PPPs are all weighted by US quantities. The Laspeyres PPPs for agriculture as a whole (last row of Table A.4) are all bigger than the Paasche PPPs (last row of Table A.3) except for the *numeraire* country, the USA. This is in line with ICP results, and reflect Gerchenkron's law.

Table A.5 shows the Geary-Khamis PPPs. For each item, the PPP is the ratio of the price of the relevant item in a country relative to the Geary-Khamis international price (both from Table A.2). Thus for Argentine wheat we divide the 1476 peso price by the $142.28 Geary-Khamis price to arrive at a PPP of 10.374 pesos to the dollar. At the basic heading level the Geary-Khamis PPP for Argentine cereals is the value (quantity times price) of the 9 individual cereal items at Argentine prices divided by their value at Geary-Khamis prices.

Value Tables

We have now described all the A and B tables and can proceed to the value tables. C.1A is B.1 times the 12th column of Table A.2. This gives values at US (plus CPD) prices, or Paasche prices, and own country weights.

C.1B is B.1 times the 13th column of Table A.1. It shows the result using the old Maddison-van Ooststroom shadow/proxy price procedure. The results are not too different from C.1A. When C.1B is divided by C.1A, we get 0.999 for Argentina and Brazil; 0.935 for India; 0.963 for Indonesia; 1.00 for Korea; 0.989 for Mexico; 0.991 for France; 0.990 for Germany; 0.982 for Japan; 1.011 for the Netherlands; 0.996 for the U.K.; and 0.999 for the U.S.A. The difference between the shadow price and CPD procedure was biggest for India and Indonesia.

C.1C is B.1 multiplied by the corresponding price array in the first 12 columns of Table A.1. When compared in Table C.1D with the total for Table C.2A (where the price holes were filled) it demonstrates our coverage (matching) ratios.

In spite of the large number of holes to be filled, the proportionate coverage of our original FAO information was generally high (93 per cent or over in all countries (except Indonesia). C.2A is the 12th column of B.1 times the first 12 columns of A.2. It shows the total value of product at national prices.

C.2B is C.2A divided by the exchange rates (shown at bottom of Table C.2. Table C.2C is C.2A divided by the Laspeyres PPPs of Table A.4. C.3 is C.2A divided by the Geary-Khamis PPPs of Table A.5 or alternatively B.1 times the Geary-Khamis price column of A.2. Tables C.4 to C.9 employ the same procedures described for C.1 to C.3 for feed and seed inputs, and for output net of feed and seed.

II The Country-Product-Dummy (CPD) Method

Historical Background

The country-product-dummy (CPD) method[2] was first proposed by Summers (1973) as a method of filling gaps in price data collected as a part of the International Comparison Program (ICP) at the University of Pennsylvania. This method was subsequently adopted as the ICP method for filling gaps in item prices prior to aggregation to the basic heading level (see Kravis, Heston and Summers (1982, pp. 87-88) for details). EUROSTAT (1982) provides a comparison of CPD and EKS[3] methods of aggregation. Cuthbert and Cuthbert (1988) discuss these methods in more detail and provide some simulation results.

Conceptual Framework

The main rationale underlying the CPD method is that the observed price of a commodity, say a potato, in a given country, say India, is the product of two components. One component is the general price level in India, relative to that of the numeraire country, and the other refers to the average (over all the countries) price of a potato.

Thus price p_{ij} of i-th item in j-th country is a product of two elements:

(i) Purchasing power parity (PPP) of the country's currency, π_j^*, which represents the number of units of the country's currency which has the same purchasing power as one unit of the numeraire currency. For example, if $\pi_j^* = 7$ and the numeraire currency is the US dollar, then 7 units of j-th currency are equal in purchasing power to one US dollar.

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2 The CPD method derives its name due to the use of product and country dummy variables in the process of regression estimation.

3 The EKS procedure is another method of aggregating item level prices with gaps, to basic heading level.

π_j^* may be interpreted as an indicator of the general price level relative to a reference/numeraire currency.

(ii) The second element is the average price of i-th commodity, denoted by η_i^*, which is an average for all the countries under consideration. It is expressed in the currency units of the reference country. If the US dollar is the reference currency unit, and $\eta_i^* = 152.0$, then the average price of i-th commodity is 152 US dollars.

CPD Model

The CPD method postulates that

$$p_{ij} = \eta_i^* \pi_j^* \tag{1}$$

For example, if i refers to wheat and j refers to USA with

$$\eta_i^* = \eta_{wheat}^* = 127.64$$

and

$$\pi_j^* = \pi_{USA}^* = 1.0$$

then

$$p_{ij} = p_{wheat,USA} = 127.64\,.$$

If the PPP for the UK is 0.65825 then

$$\begin{aligned} p_{wheat,UK} &= \eta_{wheat}^* . \pi_{UK}^* \\ &= 127.64 \,.\, 0.65825 \\ &= £84.02 \end{aligned}$$

Notes:

1. In terms of defining η_i^* and π_j^*, there is one degree of freedom. That is, we can represent the prices and purchasing power in terms of a reference currency or in terms of a numeraire commodity. In the latter case, all the prices are in the form of a price relative to the numeraire commodity. Then η_i^*, for commodities i = 1,2, ..., N, represent the average relative price structure in the countries under consideration.

2. It is unrealistic to assume that prices in different countries follow an exact/deterministic relation such as 1. So the CPD model is postulated as

$$p_{ij} = \eta_i^*.\pi_j^*.u_{ij}^* \qquad (2)$$

where u_{ij}^* is a "random" disturbance term which can account for the difference between the observed (actual) price and the expected CPD price $\eta_i^*.\pi_j^*$.

Estimation Problem

In practice η_i^* and π_j^* are unknown, but prices, p_{ij}, are observed for most of the commodities in most of the countries. The unknowns are estimated using regression-based techniques. Taking the logarithm of both sides of (2) yields

$$\log p_{ij} = \log \eta_i^* + \log \pi_j^* + \log u_{ij}$$

or

$$\boxed{\log p_{ij} = \eta_i + \pi_j + u_{ij}} \qquad (3)$$

Notes:

1. Estimates of η_i and π_j (N + M unknowns in total) can be obtained after selecting *either* a commodity price or a country's currency as a numeraire. In our estimation we use the US dollar as the common currency unit.

2. Actual estimates are obtained by regressing

$$
\textit{the observed price vector}\rightarrow
\begin{bmatrix} p_{11} \\ p_{21} \\ \cdot \\ \cdot \\ p_{N1} \\ \cdot \\ \cdot \\ p_{1j} \\ p_{2j} \\ \cdot \\ \cdot \\ p_{Nj} \\ \cdot \\ \cdot \\ p_{1M} \\ \cdot \\ \cdot \\ p_{NM} \end{bmatrix}
\quad \textit{on commodity dummies} \quad
\begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\quad \rightarrow \textit{which have } 1 \textit{ for commodity } 1 \textit{ and zeroes elsewhere} \; \cdot \quad
\begin{bmatrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 1 \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \\ 0 \end{bmatrix}
\quad \textit{and country dummies} \quad
\begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

The "CPD" method derives its name from the nature of its explanatory variables/regressors. Their peculiar characteristic is that each column consists of 1's and 0's. Hence the name "dummy variable" is used to designate these columns. Since each column corresponds to either a country or a product, this approach is therefore called the "country-product-dummy" method.

3. It is necessary to use an econometric package to estimate the unknown coefficients η_i and π_j. The output of the regression model, when the currency of country 1 is used as the reference currency, gives numerical values[4]

$$\hat{\eta}_1 \quad \hat{\eta}_2 \quad \hat{\eta}_{N \text{ or } 111} \qquad \rightarrow \text{one for each item}$$

and

$$\hat{\pi}_1 = 0 \quad \hat{\pi}_2 \ldots \hat{\pi}_{M \text{ or } 12} \qquad \rightarrow \text{one for each country.}$$

The regression package also provides a measure of reliability, known as the "standard error", associated with each estimate. If the standard error is large the derived estimate is deemed to be less reliable.

Prediction

Given $\hat{\eta}_i$ $(i = 1,2,\ldots, N)$ and $\hat{\pi}_j$ $(j = 1,\ldots,M)$.

the price of any commodity in any selected country, say p_{ij}, can be predicted using[5]

$$p_{ij} = \exp\left[\hat{\eta}_i + \hat{\pi}_j\right] = \hat{\eta}_i^* \hat{\pi}_j^*$$

where $\exp(\hat{\eta}_i) = \hat{\eta}_i^*$

and $\exp(\hat{\pi}_j) = \hat{\pi}_j^*$.

In our empirical example we have

$$\hat{\eta}_3 = 9.4085 \qquad \hat{\pi} = -3.1243$$

$\downarrow$ Barley $\qquad$ $\downarrow$ Netherlands

Then

$$\begin{aligned} \hat{p}_{37} &= \text{price of Barley in Netherlands} \\ &= \exp[9.4085 + (-3.1243)] \\ &= \exp[6.2842] \\ &= 536.04. \end{aligned}$$

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[4] Symbol ^ is used to denote that these are estimates of the true, but unknown, values η_i and π_j.

[5] symbol exp is just the opposite/inverse of logarithm function; i.e., if b = log(a) then a = exp(b).

CPD Technique for filling holes at item level

If price p_{ij} is missing then it is filled with a predicted price from the CPD model. Thus,

$$p_{ij}^{CPD} = \exp(\eta_i + \pi_j)$$

Aggregation from item level to basic heading level: the ICP procedure

1. For any pair of countries, say UK and USA, the price at the basic heading level is in the form of a PPP. Thus the PPP for the UK is given by

$$PPP_{UK} = \prod_{i=1}^{N} \left(\frac{p_{iUK}}{p_{iUSA}} \right)^{1/N}$$

= A simple geometric mean of all the price ratios within a basic heading.

Since all the price ratios are given the same weight, the resulting parities are transitive and base-invariant.

2. The basic heading PPPs together with expenditure data form the basic input into the Geary-Khamis method.

3. When basic headings have no price data, this problem is again solved using the CPD technique. Since expenditures are available at this level, a weighted CPD regression, where price observations are weighted with expenditure, is used for predicting missing PPPs at the basic heading level.

Note: To apply these procedures it is necessary to have a PPP at the basic heading level for at least two countries for the categories where there holes in the data.

III. Geary-Khamis method for multilateral comparisons

This method is the most widely used aggregation or index number method for international comparisons (see Kravis et. al, 1982 and Prasada Rao, 1993). The method draws its title from the principal contributors to the development of the method, Professors Geary and Khamis, both well-known statisticians. Geary (1958) provides the framework underlying this method based on the idea of the purchasing power parity (PPP) of a currency. This framework was further refined in Khamis (1972) where he described the many interesting mathematical and statistical properties of the method.

Let π_j represent the general price level observed in a country, which obviously depends upon the prices observed in the country. Then the price index, I_{jk}, for country k with country j as the base can be defined as:

$$I_{jk} = \frac{\pi_j}{\pi_k} \tag{1}$$

If π_js are known, then the indices can be computed. It is easy to see from equation (1) that:

(i) the index numbers in (1) are transitive; and that:

(ii) they do not change if each π_j is multiplied by the same constant.

This means that it is sufficient, for index number purposes, if the ratios of π_js are uniquely determined through use of an appropriate method.

Geary (1958) defines the purchasing power of currency j, denoted by PPP_j, as the reciprocal of the general price level in country j, π_j. Thus

$$PPP_j = \frac{1}{\pi_j}; \text{ and}$$

$$I_{jk} = \frac{\pi_j}{\pi_k} = \frac{PPP_k}{PPP_j}. \tag{2}$$

From equation (2) it is evident that if PPP_js can be determined then the necessary price index numbers can be computed.

PPP_j shows the number of currency units of j-th country currency equivalent in purchasing power to one unit of a reference or base country currency. Thus if the PPP of

Australian dollars in terms of the US dollar is Au$1.21 = $1.00, this means that 1.21 Australian dollars have the same purchasing power as 1 US dollar. Thus the PPP can be used as an alternative converter for expressing Australian value added in US dollars instead of exchange rate.

The main question then is how to measure this PPP. The Geary-Khamis method derives PPP_js using the observed price and quantity data. But the method introduces another concept known as the "*international average price*" of a commodity, denoted by P_i, for each i=1,2,...,N. These international average prices are expressed in a common currency unit or a reference currency or a *numeraire* currency.

The Geary-Khamis method determines (for M countries and N commodities):

(i) M purchasing power parities, PPP_1, PPP_2, ..., PPP_M; and

(ii) N commodity international average prices, P_1, P_2, ..., P_N.

using the observed price-quantity data.

The procedure provides an intuitively obvious set of interrelated equations to define the PPPs and the international prices.

International prices

Suppose the PPP_js are known. Then define international price of i-th commodity (i=1,2,...,N) as:

$$P_i = \frac{\sum_{j=1}^{M}\left[p_{ij}q_{ij}/PPP_j\right]}{\sum_{j=1}^{M} q_{ij}} \tag{3}$$

The denominator of equation (3) is simply the total quantity of i-th commodity in all the M countries involved in the comparisons. The numerator is the total value of i-th commodity over all the countries, after each country's value, $p_{ij}q_{ij}$ is converted into a common currency unit using respective PPPs. This is repeated for all the commodities.

Purchasing Power Parities

With the Geary-Khamis method, the purchasing power parities, PPP_j, are determined using the following equation. For country j, PPP_j, is defined as:

$$PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}} \qquad (4)$$

The numerator in equation (4) is the total of value of all the quantities in country j, expressed in the currency units of country j; and the denominator represents the value of country j's commodity bundle valued at international average prices expressed in some selected reference country (common) currency units. Thus the ratio in (4) provides a PPP for country j's currency.

Solving the Geary-Khamis System

The Geary-Khamis system consists of the (M+N) equations, (3) and (4), in the unknown entities PPP_j (j=1,2,...,M) and P_i(i=1,2,...,N). Further these equations are interdependent in that values of PPP_js depend upon international prices, P_is, which in turn depend upon the unknown purchasing power parities, PPP_js.

The Geary-Khamis system is meaningful only if a unique positive solution exists for the unknown PPP_js and P_is. This was proved in Khamis (1972) where it was shown that a solution which is positive and unique up to a factor of scalar multiplication exists for the unknowns in the system.

. Thus Khamis proved that if one of the PPP_js is set to unity, then rest of the unknown parities and international prices can be *uniquely solved.* This offers a choice as to which country's currency is set to unity. If PPP of country 1's currency is set to unity, then PPP's of all other currencies expressed in terms of country 1's currency. Similarly all the international prices give international average price of different commodities expressed in country 1's currency.

In most empirical studies, the US dollar is used as the reference currency for which the PPP is set to unity. However, since the PPP's are unique up to a factor of proportionality, ratios of the form PPP_j/PPP_k are independent of the choice of the currency selected.

Now we outline a simple method to solve the Geary-Khamis equations.

Iterative Method

This method is an intuitive procedure based on the circular nature of the equations (3) and (4). The following steps are involved:

Step 1: Start with any positive values for PPP_1, PPP_2, ..., PPP_M with one selected currency unit as the reference currency. If country 1 is the reference currency, then we choose any set of positive values with $PPP_1 = 1.0$. The most obvious starting point could be to set all PPP_j's to be unity.

Step 2: Use the starting values of PPP_j's in equations (3) and compute international average prices. Use these resulting international prices to compute the next round purchasing power parities using equation (4).

Then repeat Steps 1 and 2 until the values converge, making sure that at each stage the PPP's obtained are normalized to make $PPP_1 = 1.0$.

Convergence and uniqueness

This iterative procedure is useful provided it converges and converges to the same values irrespective of the starting values used. Khamis (1972) established viability of the iterative procedure just outlined.

In fact, the speed of convergence is amazingly fast. Even with a large number of countries, the procedure converges in 10 to 15 iterations.

A numerical illustration

The following numerical example illustrates the multilateral comparison problem for three countries, USA, India and Brazil. Let us consider a purely hypothetical array of price and quantity information for wheat, potatoes, milk and lamb meat.

Item	USA ($)		India (Rs)		Brazil (Cruzeiros)	
	Price	Qty.	Price	Qty.	Price	Qty.
Wheat	1.42	30	22	20	4	26
Potatoes	1.1	50	17	26	3.2	42
Milk	2.85	120	40	43	8	80
Lamb	24	140	380	30	80	45
total value	3800		14002		4478.4	

<u>Step 1:</u>

Starting values of PPP_j's:

$PPP_1 = 1.0 \quad PPP_2 = 1.0 \quad PPP_3 = 1.0$

<u>Step 2:</u>

International Prices from equation (3) with the starting values are:

$P_1 = 7.72$ $P_2 = 5.35$ $P_3 = 11.12$ $P_4 = 85.40$

The next set of PPP's are calculated using (4) and the above international prices are:

$PPP_1 = 0.2756$ $PPP_2 = 4.2004$ $PPP_3 = 0.8683$

After normalizing these parities so that $PPP_1 = 1.0$ (divide all PPPs by 0.2756) then

$PPP_1 = 1.00$ $PPP_2 = 15.24$ $PPP_3 = 3.15$

Step 3:

In the next step, international prices, using the new normalized parities, are:

$P_1 = 1.37$ $P_2 = 1.07$ $P_3 = 2.71$ $P_4 = 2.67$

These international prices can be used in deriving the next step parities by substituting these prices into (4). Then

$PPP_1 = 0.9898$ $PPP_2 = 15.48$ $PPP_3 = 3.21$

After normalizing these parities so that $PPP_1 = 1.0$ (divide all PPPs by 0.9898) then

$PPP_1 = 1.00$ $PPP_2 = 15.64$ $PPP_3 = 3.24$

In the next step the PPPs converge. The final values of purchasing power parities and international prices with US dollar as the reference currency are:

PPP's:

\$1.0 = US\$1.0 \$1.0 = Rs.15.64 \$1.0 = Cruzeiro 3.24

International Prices:

Wheat = \$1.35 Potatoes = \$1.06 Milk = \$2.67 Lamb = \$24.18.

Properties of the Geary-Khamis Method

The following is a list of properties of the Geary-Khamis method which can be proved using simple algebra.

(1) The price index numbers underlying the Geary-Khamis method are defined simply as the ratios of the purchasing power parities. Index for country k with country j as the base is defined as:

$$I_{jk}(\text{Price}) = \frac{PPP_k}{PPP_j} \tag{5}$$

It is easy to check that the price indices in (5) are transitive and base invariant. The price index from the Geary-Khamis system can be derived as an algebraic expression when the number of countries involved in the problem is equal to 2, i.e., M = 2. Then using simple algebra we can show that

$$I_{12}(\text{Price}) = \frac{\sum_{i=1}^{N} p_{12} \frac{q_{i2} q_{i1}}{q_{i2} + q_{i1}}}{\sum_{i=1}^{N} p_{11} \frac{q_{i2} q_{i1}}{q_{i2} + q_{i1}}}$$

In this case there is no need to compute the purchasing power parities separately.

(2) The quantity index numbers are defined as:

$$I_{jk}(\text{Quantity}) = \frac{\sum_{i=1}^{N} P_i q_{ik}}{\sum_{i=1}^{N} P_i q_{ij}} \tag{6}$$

Quantity index numbers in equation (6) are also transitive. An intuitive interpretation of the quantity index number in (6) is that it is a ratio of the quantities in countries k and j valued at a common set of international prices, P_i.

(3) The price and quantity index numbers, defined respectively in equations (5) and (6) satisfy the factor test that:

$$I_{jk}(\text{Price}) \times I_{jk}(\text{Quantity}) = \frac{\sum_{i=1}^{N} p_{ik} q_{ik}}{\sum_{i=1}^{N} p_{ij} q_{ij}} = \frac{\text{Value in country k}}{\text{Value in country j}}$$

(4) The Geary Khamis international prices and purchasing power parities satisfy the property that:

$$\frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{PPP_j} = \sum_{i=1}^{N} P_i q_{ij} \qquad (7)$$

The right-hand-side of equation 7 represents the value of quantities in country j at international prices, expressed in a common currency unit, whereas the left-hand-side represents the value in country j converted into a common currency unit using the PPP for the country. The Geary-Khamis method guarantees the same value aggregate whether obtained through a currency conversion of the total value or through a revaluation of country commodity bundle at international average prices. This is generally referred to as the property of *additive consistency*.

Conclusion

In view of these excellent properties the Geary-Khamis method was selected in the 1970's as the principal aggregation/index number procedure for use in the International Comparisons Project (ICP) of the United Nations. It is also the main method used in international comparisons of agricultural production aggregates (see Prasada Rao, 1993) by the FAO in Rome. OECD uses the procedure in deriving purchasing power parities of currencies of its member countries (see OECD, 1990).

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